CS275 GRADED HOMEWORK 4 - SOLUTION

GIVE BACK ON TUESDAY OCTOBER. 19TH 2004 AT BEGINNING OF CLASS

For each question, read **each word** with the greatest care and **without hurrying**. If you have doubts about what is asked, **go back** to the wording of the question until the meaning of the question is clear. Then try to find an answer. If you get stuck, don't hesitate to **contact** your T.A. or me.

Please write your section number on your homework as well as a rough estimate of the time you spent solving it.

Exercise 1. In town, there are just 3 restaurants:

- A seafood restaurant serving dishes from a "set" $\mathcal{S}$.
- A vegetarian restaurant serving dishes from a "set" $\mathcal{V}$. $\mathcal{S} \cap \mathcal{V} = \emptyset$.
- A carnivorous restaurant serving dishes from a "set" $\mathcal{C}$. $\mathcal{S} \cap \mathcal{C} = \mathcal{V} \cap \mathcal{C} = \emptyset$.

A dinner consists of one dish or two dishes (the order matters) from the same restaurant and ends with either a cup of coffee, a cup of liquor, a mint or nothing. Let $\mathcal{E} = \{\text{Coffee, Liquor, Mint, Nothing}\}$ be the set of possible endings.

a) Write in mathematical notation the set of possible dinners.
b) What is its cardinal? I.e. in how many different ways can one have dinner?
c) What is the cardinal of the set of dinners in which two distinct dishes are served?

Exercise 2. Exercise 8 p. 109 of [1]: Find these values.

a) $\lfloor 1.1 \rfloor$
b) $\lceil 1.1 \rceil$
c) $\lfloor -0.1 \rfloor$
d) $\lceil -0.1 \rceil$
e) $\lceil 2.99 \rceil$
f) $\lceil -2.99 \rceil$
g) $\lfloor \frac{1}{2} + \lceil \frac{1}{2} \rceil \rfloor$
h) $\lceil \lfloor \frac{1}{2} \rfloor + \lceil \frac{1}{2} \rceil + \frac{1}{2} \rceil$

Exercise 3. For each of the functions below, determine whether it is onto and/or one-to-one.

a) $f : n \in \mathbb{N} \longrightarrow n + 1 \in \mathbb{N}$
b) $f : x \in \mathbb{Z} \longrightarrow x^3 \in \mathbb{Z}$
c) $f : x \in \mathbb{R} \longrightarrow x^3 \in \mathbb{R}$
d) $f : n \in \mathbb{Z} \longrightarrow \lceil n/2 \rceil \in \mathbb{Z}$
e) $f : n \in \mathbb{N} \longrightarrow (-1)^n \lfloor n/2 \rfloor \in \mathbb{Z}$

Exercise 4. Let $A = \{1 \ldots 10\}$ and $B = \{1 \ldots 20\}$ and define the functions

$$\begin{array}{llll} f: & x \in A & \longrightarrow & 2x \in B \\ g: & x \in B & \longrightarrow & \lceil \frac{x}{2} \rceil \in A \end{array}$$

a) Prove or disprove that f is onto and that it is one-to-one.
b) Prove or disprove that g is onto and that it is one-to-one.
c) Prove or disprove that $g \circ f$ is onto and that it is one-to-one.

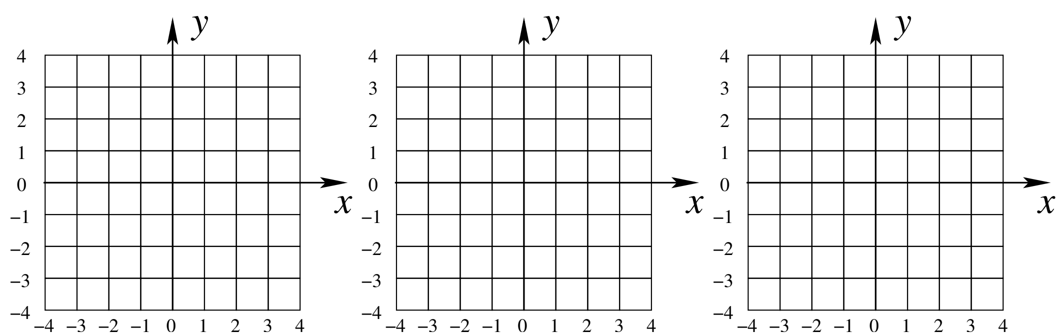
Exercise 5. Let R be a relation on a set A containing two or more elements. Prove or disprove the following statements

a) If R is symmetric and transitive, then R is reflexive.
b) If R is an equivalence relation, then R is not a total order relation.
c) If R is an equivalence relation, then R is not a partial order relation.
d) If R is symmetric and antisymmetric, then $\forall x,\, y \in A,\ R(x,y) \Longrightarrow x = y$.

Exercise 6. For each of the relations below

a) $A = \mathbb{Z},\ R(x,y) \equiv x = \max\{x,y\}$
b) $A = \mathbb{Z},\ R = \left\{(x,y) \mid x \le y \wedge x^2 - y^2 = 0\right\}$
c) $A = \mathbb{Z},\ R = \left\{(x,y) \mid x \le y \vee x^2 - y^2 = 0\right\}$

1): Circle, on the grids[1] $\{-4,...,4\} \times \{-4,...,4\}$ of Figure 6.1, the points (x,y) that verify $R(x,y)$.
2): Prove or disprove that R is reflexive, symmetric, antisymmetric, transitive. Don´t forget that the R is defined on $\mathbb{Z}$, not just $\{-4,\dots,4\}$.



(a) (b) (c)

FIGURE 6.1. Grids for Exercise 6. Don´t forget that the relations are defined on $\mathbb{Z}$ and not just $\{-4\dots4\}$. These grids are here only to illustrate "what the relations look like".

Exercise 7. Let $A = \{0,1\}^3 \cup \{0,1\}^4 \cup \{0,1\}^5 \dots$ be the set of bit strings of length 3 or more and let R be the relation on A consisting of all pairs (x,y) such that x and y are equal except perhaps in their first three bits.

a) Show that R is an equivalence relation on A.
b) Write in mathematical notation the equivalence classes of the strings
  1) 110
  2) 1010
  3) 11110
c) Write in lexicographic order 4 elements of each of the equivalence classes above.

Note: This exercise is largely inspired by Exercises 8 p. 513 and 25 p. 514 of [1].

REFERENCES

[1] K. H. Rosen. *Discrete Mathematics and Its Applications*. Mc Graw Hill, 5 edition, 2003.

[1] You may copy these grids on any sheet of paper.