WEEK 10 : CS275 RECITATION EXERCISES

For each question, read **each word** with the greatest care and **without hurrying**. If you have doubts about what is asked, **go back** to the wording of the question until the meaning of the question is clear. Then try to find an answer.

Exercise 1. Use inverters, AND and OR gates to build circuits with these outputs:

a) $(x+y)(x+z)$
b) $x+yz$
c) $xyz+\bar{x}\bar{y}\bar{z}$
d) $(x+\bar{y})(y+\bar{z})(z+\bar{x})$

Exercise 2. Build circuits using only NAND gates, to produce these outputs:

a) $\bar{x}$
b) xy
c) $x+y$
d) $x(y+\bar{z})$

Exercise 3. Determine whether the XOR, NAND and NOR operators ($\oplus$, $|$ and $\downarrow$) are associative, i.e. if, for all x, y and z in $\{0,1\}$, one has $x\oplus(y\oplus z)=(x\oplus y)\oplus z$, $x|(y|z)=(x|y)|z$ and $x\downarrow(y\downarrow z)=(x\downarrow y)\downarrow z$. Does it make sense to write, then $x\oplus y\oplus z$, $x|y|z$ and $x\downarrow y\downarrow z$ (or to build three-input XOR, NAND [1] and NOR gates)?

Exercise 4. Solve Exercise 10 p. 718 of [1]. Construct a circuit for a half subtractor using AND gates, OR gates and inverters. A **half subtractor** has two bits as input and produces as output a difference bit and a borrow.

Hint: You may want to build the truth table of $x-y$ and the truth table of the borrow bit.

Exercise 5. Solve Exercise 8 p. 423 of [1]. A model for the number of lobsters caught per year is based on the assumption that the number of lobsters caught in a year is the average of the number caught in the two previous years.

a) Find a recurrence relation for $\{L_n\}$, where L_n is the number of lobsters caught in year n, under the assumption for this model.
b) Find L_n if $100,000$ lobsters are caught in year 1 and $300,000$ were caught in year 2.

Exercise 6. Find the solutions to the homogeneous recurrence relations and compute the first few terms of the sequence

a) $x_n=2x_{n-1}-x_{n-2}$ with initial conditions $x_0=2$ and $x_1=-4$
b) $x_n=x_{n-1}+\frac{3}{4}x_{n-2}$ with initial conditions $x_0=-2$ and $x_1=1$
c) $x_n=x_{n-1}+\frac{3}{4}x_{n-2}$ with initial conditions $x_0=-\frac{3}{2}$ and $x_1=1$

References

[1] K. H. Rosen. *Discrete Mathematics and Its Applications.* Mc Graw Hill, 5 edition, 2003.

[1] Note: there exist 3-input NAND and NOR gates (e.g. 74LS10 and 74LS27), which output $\overline{xyz}$ and $\overline{x+y+z}$.