CS275 WEEK 12 RECITATION EXERCISES

For each question, read **each word** with the greatest care and **without hurrying**. If you have doubts about what is asked, **go back** to the wording of the question until the meaning of the question is clear. Then try to find an answer. If you get stuck, **contact** your T.A. or me.

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1. Reminders from two weeks ago

1.1. Big-O, small-o.
This is not really part of the syllabus, but may be of use.

A function g defined on the natural numbers is said to **asymptotically bound** a function f if, when n is big, $f(n)$ is not greater than a constant times $g(n)$. One then writes

$$f(n) \in O(g(n)).$$

Using a more mathematical notation, this definition is : There exist positive constants C and N such that, if $n > N$, one has

$$f(n) < C\,g(n).$$

A function g is said to **asymptotically dominate** a function f if, when n is big, $\frac{f(n)}{g(n)}$ tends to zero. One then writes

$$f(n) \in o(g(n)).$$

This definition, in mathematical notation, says that $\frac{f(n)}{g(n)}$ can be made as small as one wants, when one increases n. More precisely, for any (small) $\varepsilon > 0$, and if n is big enough, one has $\frac{f(n)}{g(n)} < \varepsilon$. This is written

$$\forall \varepsilon > 0,\ \exists N \in \mathbb{N},\ \forall n > N,\ \frac{f(n)}{g(n)} < \varepsilon.$$

Here, N is the point at which n is "big enough."

2. Exercises

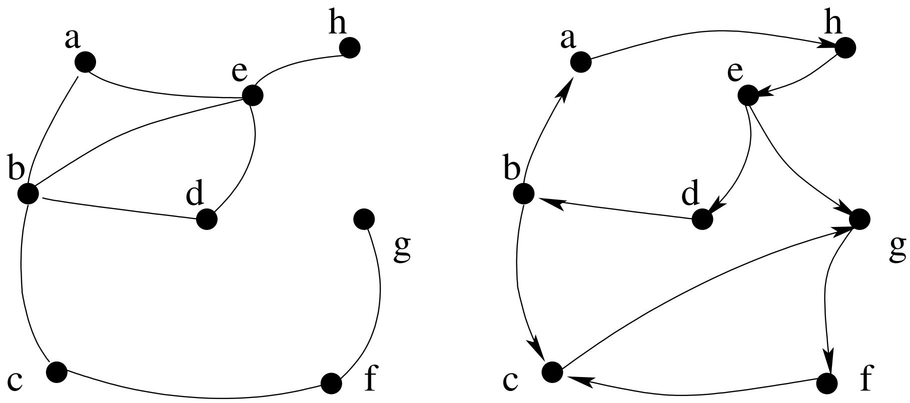
Exercise 1. Solve Exercise 12 p. 555 of [1]. What does the degree of a vertex represent in the acquaintanceship graph, where vertices represent all the people in the world? What do isolated and pendant vertices in this graph represent? In one study it was estimated that the average degree of a vertex in this graph is 1000. What does this mean in terms of the model?

Exercise 2. Solve Exercise 28 p. 555 of [1]. Does there exist a simple graph with six vertices of these degrees? If so, draw such a graph.

(a) 0,1,2,3,4,5	**(b)** 1,2,3,4,5,6	**(c)** 2,2,2,2,2,2	**(d)** 3,2,3,2,3,2
(e) 3,2,2,2,2,3	**(f)** 1,1,1,1,1,1	**(g)** 3,3,3,3,3,5	**(h)** 1,2,3,4,5,5

Exercise 3. Draw the call graph as a directed graph, if

- Alice called Bob,
- Bob called Alice,
- Bob called Calvin,
- Calvin called Bob,
- Alice called Mallet,
- Donald called Alice,
- Calvin called Donald and
- Mallet called Calvin.



(a) **(b)**

Figure 2.1. **(a)** Graph for Ex. 5. **(b)** Graph for Ex. 5

Write in mathematical notation the sets of vertices and edges of this graph.

Exercise 4. Consider the simple graph of Figure 2.1, (a). Write down the following elements:

- Vertices, edges.
- Degree of each vertex.
- Isolated and pendant vertices.
- Connected components of the graph.

In the same figure, find, if possible

- A path starting at h, passing through a and ending at g.
- Two simple paths starting at h, passing through d and ending at g.
- A simple cycle starting at a and passing through g.
- A cycle starting at h and passing through g.

Exercise 5. Consider the directed graph of Figure 2.1, (b). What are:

- The vertices, edges.
- The in-degree, out-degree and degree of each vertex.
- Isolated and pendant vertices.
- Weakly and strongly connected components of the graph.

References

[1] K. H. Rosen. *Discrete Mathematics and Its Applications*. Mc Graw Hill, 5 edition, 2003.