CS275 WEEK 13 RECITATION EXERCISES - PROVISIONAL SOLUTION

For each question, read **each word** with the greatest care and **without hurrying**. If you have doubts about what is asked, **go back** to the wording of the question until the meaning of the question is clear. Then try to find an answer.

1. Reminders

2. Exercises

Exercise 1. Determine which of the pairs of graphs in Figure 1.1 are isomorphics. To show that they are isomorphic, find an isomorphism. To show that they are not, provide a proof.

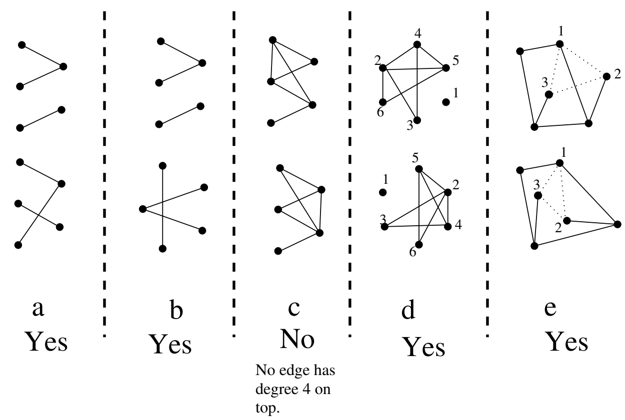

FIGURE 1.1. Find isomorphisms (Ex. 1)

Exercise 2. Solve Exercise 54 p. 566 of [1]. How many nonisomorphic simple graphs of are there with n vertices, when n is

Solution:
See Fig. 2.1.

Exercise 3. Solve Exercise 26 p. 590 of [1]. For which values of n do these graphs have an Euler circuit?

Solution:

a) Odd n. Each vertex of K_n has degree $n-1$, which is even iff n is odd.

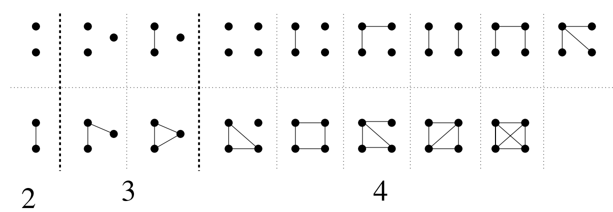

FIGURE 2.1. Find isomorphisms (Ex. 2)

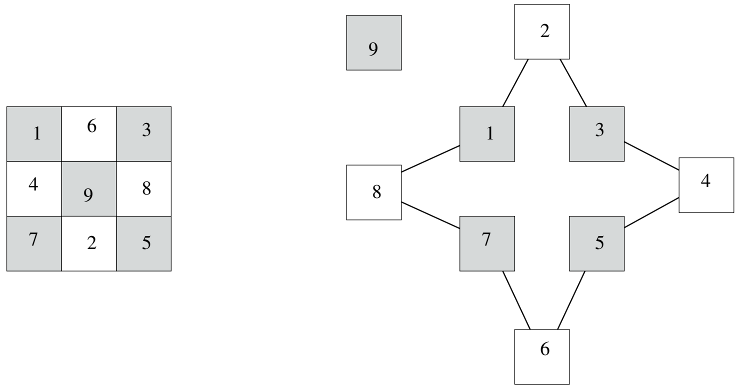

FIGURE 4.1. 3×3 chess board and the graph representing the possible moves of a knight.

b) All. Each vertex of C_n has degree 2, for all n.
c) None. Each vertex on the "wheel" part of W_n has degree 3, for all n.
d) Even n. Each vertex of Q_n has degree n.

Exercise 4. Recall the legal movement of a knight on a chess board. This exercise is about modelling a 3×3 chess board and the possible moves of a knight on it.
Let V be the set of squares on a 3×3 chess board. Let E be the set of (unordered) pairs of squares s.t. a knight can go from one to the other in a single move.

a) Write down V and E.
Solution: Take for example $V = \{1 \ldots 9\}$ as in Fig. 4.1 and $E = \{\{1,2\}, \{2,3\}, \ldots, \{7,8\}, \{8,1\}\}$.

b) Is (V, E) connected?
Solution: No. 9 is an isolated vertex.

c) Is (V, E) a planar graph?
Solution: Yes. See Fig. 4.1

Exercise 5. What is the chromatic number of the following graphs?

Solution:

a) $n - 1$.
b) 2.
c) 2 for even n, 3 for odd n.
d) 2. One color for vertices w/ even number of 1 bits, one color for vertices w/ odd number of 1 bits. Each vertex can be represented be a sequence of n bits.
e) 3 for even n, 4 for odd n. Same as the wheel, plus one.

REFERENCES

[1] K. H. Rosen. *Discrete Mathematics and Its Applications.* Mc Graw Hill, 5 edition, 2003.