CS275 RECITATION EXERCISES

For each question, read **each word** with the greatest care and **without hurrying**. If you have doubts about what is asked, **go back** to the wording of the question until the meaning of the question is clear. Then try to find an answer.

Be careful with notation and words:

$\{1,2,3\}$ is a **set** with elements 1, 2 and 3. The order does not matter, this is the same thing as $\{3,2,1\}$, $\{3,1,2\}$, $\{1,3,2\}$, $\{2,3,1\}$ and $\{2,1,3\}$.

When the elements in a set are all natural numbers, a set can also be called a **combination**. For example, the set $\{1,2,3\}$ is a combination of 3 out of the 9 elements of the set $\{1,...,9\}$.

$(1,2,3)$ is a **sequence** whose first element is 1, second is 2 and third is 3. The order does matter. This is not the same as $(2,1,3)$. This is sometimes called a **permutation with repetition**.

When the elements in a sequence are all distinct natural numbers, this sequence is also a **permutation**. $(3,2,1)$ is a permutation. $(2,1,2)$ is not.

Exercise 1. List all the permutations of two elements of the set $\{0,1,2\}$? List them in lexicographic order.

Exercise 2. List all the subsets of two elements of the set $\{0,1,2\}$? List them in lexicographic order.

Exercise 3. List all the sequences of two elements of the set $\{0,1,2\}$? List them in lexicographic order.

Exercise 4. List all the increasing sequences of two elements of the set $\{0,1,2\}$? List them in lexicographic order. Any resemblence with one of the cases above?

Exercise 5. For each of the statements below, find an object X that verifies it. Which of these statements are equivalent?

- **a)** X is a permutation of p out of n elements[1].
- **b)** X is a subset of p out of n elements.
- **c)** X is a permutation of p out of n elements with repetition.
- **d)** X is a sequence of p elements of the set $\{1...n\}$.
- **e)** X is a sequence of p distinct elements of the set $\{1...n\}$.
- **f)** X is a strictly increasing sequence of p elements of the set $\{1...n\}$.

Exercise 6. What is the number of 5-element subsets of $\{0...7\}$?

[1] By default, permutation is "without repetition."

Exercise 7. Let $(x_1, ..., x_5)$ be a sequence, where $x_i \in \{0...7\}$ for all i.

a) Write in mathematical notation the set of these sequences.
How many different sequences are there?

b) Write in mathematical notation the set of sequences $(x_1, ..., x_5)$ where the x_i are two-by-two distinct.
How many such sequences are there?

c) Write in mathematical notation the set of sequences $(x_1, ..., x_5)$ where $x_1 < ... < x_5$.
How many such sequences are there?

Exercise 8. Solve Exercise 14 p. 310 of the textbook [1]: how many distinct bit strings of length n, where n is a positive integer, start and end with 1s?

Exercise 9. Solve Exercise 40 p. 312 of the textbook [1]: how many bit strings of length 7 either begin with two 0s or end with three 1s?
Hint: Inclusion-exclusion principle.

Exercise 10. Solve Exercise 14 p. 342 of the textbook [1]: how many solutions are there to the equation

$$x_1 + x_2 + x_3 + x_4 = 17,$$

where x_1, x_2, x_3 and x_4 are nonnegative integers?
Hint: In Tuesday Sep. 21st class, x_i were number of fishes caught.

Exercise 11. Solve Exercise 6 p. 333 of the textbook [1]: what is the coefficient of x^7 in $(1 + x)^{11}$?
Hint: Binomial theorem.

Exercise 12. Solve Exercise 8 p. 333 of the textbook [1]: what is the coefficient of x^8y^9 in the expansion of $(3x + 2y)^{17}$?

Exercise 13. Solve Exercise 16 p. 342 of the textbook [1]: how many solutions are there to the equation

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 29,$$

where x_i, for all $i \in \{1, 2, 3, 4, 5, 6\}$ is a nonnegative integer such that

a) $x_i > 1$, for all $i \in \{1, 2, 3, 4, 5, 6\}$?
b) $x_i \geq i$, for all $i \in \{1, 2, 3, 4, 5, 6\}$?
c) $x_1 \leq 5$?
d) $x_1 < 8$ and $x_2 > 8$?

Exercise 14. Solve Exercise 10 p. 253 of the textbook [1]: prove that $1{\cdot}1! + 2{\cdot}2! + \cdots + n{\cdot}n! = (n + 1)! - 1$ whenever n is a positive integer.

References

[1] K. H. Rosen. *Discrete Mathematics and Its Applications.* Mc Graw Hill, 5 edition, 2003.