

CS275 RECITATION EXERCISES

For each question, read **each word** with the greatest care and **without hurrying**. If you have doubts about what is asked, **go back** to the wording of the question until the meaning of the question is clear. Then try to find an answer.

Be careful with notations and words:

$f : \mathbb{R} \rightarrow \mathbb{N}$ reads

“ f is a function mapping \mathbb{R} into \mathbb{N} ” or

“ f is a function from \mathbb{R} to \mathbb{N} ” or

“ f maps \mathbb{R} into \mathbb{N} .”

$f : x \in \mathbb{R} \rightarrow \lfloor x \rfloor \in \mathbb{Z}$ reads

“ f maps any real number x to the integer “floor x ” or

“ f is the function that associates “floor x ” to a real number x . (in addition, its co-domain is \mathbb{Z} .”

$[a, b[= [a, b) = \{x \in \mathbb{R} \mid a \leq x < b\}$ reads

“the interval a, b , closed on one side and open on the other.”

Exercise 1. Let f be the function.

$$f : x \in]0, +\infty[\rightarrow 2 \log(x) \in \mathbb{R}$$

- What are the domain and co-domain of f ?
- Write in mathematical notation the range of f .
- Find three elements in the range of f .
- What is the image of 10 by f ?
- Find a pre-image of 10 by f .

Exercise 2. Let g be the function.

$$g : x \in \mathbb{N} \rightarrow \sqrt{x} \in \mathbb{R}$$

- What are the domain and co-domain of g ?
- Write in mathematical notation the range of g .
- Find three elements in the range of g .
- What is the image of 10 by g ?
- Find a the pre-image of 10 by g .
- Find a pre-image of $1/2$ by g .

Exercise 3. Let h be the function.

$$h : (n, x) \in \mathbb{N} \times \mathbb{R} \rightarrow 1 + x + \dots + x^n \in \mathbb{R}$$

- What are the domain and co-domain of h ?
- Write in mathematical notation the range of h .
- Find three elements in the range of h .
- What is the image of $(2, 10)$ by h ?
- Find a pre-image of 10 by h .

Exercise 4. Let r be the function.

$$r \begin{cases} \mathbb{Z} \times \mathbb{R} & \longrightarrow & \mathbb{R} \\ (m, x) & \longrightarrow & 1 + x + \dots + x^m \end{cases}$$

Compare this function to the previous function.

Exercise 5. Let s be the function.

$$s : u \in \{0..99\} \longrightarrow \left(\left\lfloor \frac{u}{10} \right\rfloor, u - 10 \cdot \left\lfloor \frac{u}{10} \right\rfloor \right) \in \mathbb{N} \times \mathbb{N}$$

- a) What are the domain and co-domain of s ?
- b) Write in mathematical notation the range of s .
- c) Find three elements in the range of s .
- d) What is the image of 10 by s ?
- e) Find a pre-image of $(7, 6)$ by s .

Exercise 6. What is wrong in the following function definition?

$$f : x \in \mathbb{N} \longrightarrow \sqrt{x} \in \mathbb{N}$$

Exercise 7. For each of the following functions,

- a) Either prove that it is onto or prove that it is not onto.
- b) Either prove that it is one-to-one or prove that it is not one-to-one.
- c) Either prove that it is a bijection or prove that it is not a bijection.

$$\begin{aligned} f : x \in \mathbb{R}^+ &\longrightarrow \sqrt{x} \in \mathbb{R} \\ g : x \in \mathbb{R}^+ &\longrightarrow \sqrt{x} \in \mathbb{R}^+ \\ h : x \in \mathbb{N} &\longrightarrow \sqrt{x} \in \mathbb{R}^+ \\ r : x \in \mathbb{R} &\longrightarrow (x, 2x) \in \mathbb{R} \times \mathbb{R} \\ s : x \in \mathbb{R} &\longrightarrow 2x \in \mathbb{R} \\ t : x \in \mathbb{N} &\longrightarrow 2x \in \mathbb{N} \end{aligned}$$

REFERENCES