

WEEK 7 : CS275 RECITATION EXERCISES

For each question, read **each word** with the greatest care and **without hurrying**. If you have doubts about what is asked, **go back** to the wording of the question until the meaning of the question is clear. Then try to find an answer.

Exercise 1. P. 480, n. 3 in [1], Section 7.1. For each of these relations on the set $\{1, 2, 3, 4\}$, decide whether it is reflexive, whether it is symmetric, whether it is antisymmetric, and whether it is transitive.

- a) $\{(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)\}$
- b) $\{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)\}$
- c) $\{(2, 4), (4, 2)\}$
- d) $\{(1, 2), (2, 3), (3, 4)\}$
- e) $\{(1, 1), (2, 2), (3, 3), (4, 4)\}$
- f) $\{(1, 3), (1, 4), (2, 3), (2, 4), (3, 1), (3, 4)\}$

Exercise 2. P. 480, n. 4 in [1], Section 7.1: Determine whether the relation R on the set of all people is reflexive, symmetric, antisymmetric, and/or transitive, where $(a, b) \in R$ iff

- a) a is taller than b .
- b) a and b were born on the same day.
- c) a has the same first name as b .
- d) a and b have a common grandparent.

Exercise 3. P. 480, n. 5 in [1], Section 7.1: Determine whether the relation R on the set of all Web pages is reflexive, symmetric, antisymmetric and/or transitive, where $(a, b) \in R$ iff

- a) everyone who has visited Web page a has also visited Web page b .
- b) there are no common links found on both Web page a and Web page b .
- c) there is at least one common link found on both Web page a and Web page b .
- d) there is a Web page that includes links to both Web page a and Web page b .

Exercise 4. P. 480, n. 6 in [1], Section 7.1: Determine whether the relation R on the set of all real numbers is reflexive, symmetric, antisymmetric and/or transitive, where $(x, y) \in R$ iff

- a) $x + y = 0$.
- b) $x = \pm y$.
- c) $x - y$ is a rational number.
- d) $x = 2y$.
- e) $xy \geq 0$.
- f) $xy = 0$.
- g) $x = 1$.
- h) $x = 1$ or $y = 1$.

Exercise 5. P. 513, n. 1 in [1], Section 7.5: Which of these relations on the set $\{0, 1, 2, 3\}$ are equivalence relations? Determine the properties of an equivalence relation that the others lack.

- a) $\{(0, 0), (1, 1), (2, 2), (3, 3)\}$
- b) $\{(0, 0), (0, 2), (2, 0), (2, 2), (2, 3), (3, 2), (3, 3)\}$
- c) $\{(0, 0), (1, 1), (1, 2), (2, 1), (2, 2), (3, 3)\}$
- d) $\{(0, 0), (1, 1), (1, 3), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$
- e) $\{(0, 0), (0, 1), (0, 2), (1, 0), (1, 1), (1, 2), (2, 0), (2, 2), (3, 3)\}$

Exercise 6. P. 514, n. 20 in [1], Section 7.5: What the equivalence classes of the equivalence relations in the previous exercise.

- a) $\{(0, 0), (1, 1), (2, 2), (3, 3)\}$
- c) $\{(0, 0), (1, 1), (1, 2), (2, 1), (2, 2), (3, 3)\}$

Exercise 7. P. 513, n. 2 in [1], Section 7.5: For each of these relations on the “set” of all people are equivalence relations? Determine the properties of an equivalence relation that the others lack.

- a) $\{(a, b) \mid a \text{ and } b \text{ are the same age}\}$
- b) $\{(a, b) \mid a \text{ and } b \text{ have the same parents}\}$
- c) $\{(a, b) \mid a \text{ and } b \text{ share a common parent}\}$
- d) $\{(a, b) \mid a \text{ and } b \text{ have met}\}$
- e) $\{(a, b) \mid a \text{ and } b \text{ speak a common language}\}$

Exercise 8. P. 514, n. 21 in [1], Section 7.5: What the equivalence classes of the equivalence relations in the previous exercise.

Exercise 9. P. 512, n. 4 in [1], Section 7.5: Define three equivalence relations on the set of students in your discrete mathematics class different from the relations discussed in the text. Determine the equivalence classes of these equivalence relations.

Exercise 10. P. 527, n. 1 in [1], Section 7.6: Which of these are posets?

- a) $(\mathbb{Z}, =)$
- b) (\mathbb{Z}, \neq)
- c) (\mathbb{Z}, \geq)
- d) (\mathbb{Z}, \dagger)

Exercise 11. P. 528, n. 11 in [1], Section 7.6: Find the lexicographic ordering of these n -tuples:

- a) $(1, 1, 2), (1, 2, 1)$
- b) $(0, 1, 2, 3), (0, 1, 3, 2)$
- c) $(1, 0, 1, 0, 1), (0, 1, 1, 1, 0)$

REFERENCES

[1] K. H. Rosen. *Discrete Mathematics and Its Applications*. Mc Graw Hill, 5 edition, 2003.